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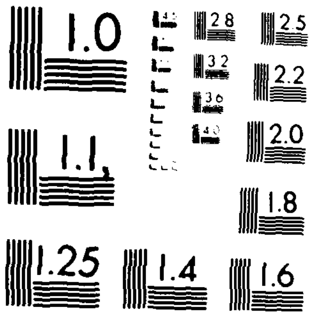


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# CENTRAL CONFIGURATIONS OF THE N-BODY PROBLEM

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# CENTRAL CONFIGURATIONS OF THE N-BODY PROBLEM

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ABSTRACT

In this paper we use topological methods, in particular Morse theory, to study the problem of finding spatial central configurations of the N-body problem in $\mathbf{R}^3$. The principal difficulty in applying Morse theory is that the potential function is defined on a manifold on which there is the action of a group which is not free. This suggests using the equivariant homology functor in order to obtain the Morse inequalities which enables us to obtain an estimate of the minimal number of spatial central configurations.

AMS (MOS) Subject Classifications: 58E05, 70F10, 70F15, 70G25

Key Words: N-Body problem, Central configurations, Morse theory, Equivariant homology, Classifying spaces.

Work Unit Number 1 - Applied Analysis

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## SIGNIFICANCE AND EXPLANATION

An important problem in celestial mechanics is to find the central configurations of the N-body problem. This problem is equivalent to looking for critical points of the relevant potential function over a manifold on which a group of symmetries acts.

The so-called collinear problem is well understood. While many important results have been obtained about the N-body problem in the plane, as far as ~~we~~ know there are no results about this problem in space.

In this paper ~~we~~ use topological methods, in particular Morse theory and the equivariant homology, to obtain a first estimate on the minimal number of spatial central configurations.

Then, using known results for the collinear and planar problem we improve this estimate and ~~we~~ are able to give an inferior bound on the number of those central configurations which are not planar in the sense that not all the bodies lie on the same plane.



The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

## CENTRAL CONFIGURATIONS OF THE N-BODY PROBLEM

Filomena Pacella

### - INTRODUCTION -

In this paper we offer a first approach to the problem of finding spatial central configurations of N bodies.

It is known, ([9], [11]) that, if $q_1,\dots,q_N$ denote the positions of N bodies with masses $m_1,\dots,m_N$ respectively, this problem is equivalent to finding the critical points of the potential energy:

$$V(q) = -\sum_{i<j} \frac{m_i m_j}{|q_i - q_j|}$$

restricted to a particular manifold.

When the bodies are on the same line this problem has been studied by F. R. Moulton who found that, for each value of $m = (m_1,\dots,m_N) \in R_+^N$, there are exactly $\frac{N!}{2}$ collinear central configurations.

Regarding the planar problem there are many interesting results obtained by J. I. Palmore using Morse theory ([5], [6], [7]).

More precisely, studying the homology of the configuration space, he finds an estimate of the minimal number of critical points that $V$ owns, whenever $m \in R_+^N$ is such that the corresponding potential energy is a Morse function, that is its critical points are non degenerate.

He explores also the case when $V$ is degenerate and he proves that $V(q)$ is a Morse function for almost every $m \in R_+^N$. Moreover he examines some

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particular cases, like that of equal masses, computing exactly the number of relative equilibria.

In our paper we consider the general case when the bodies are on the space and are not bound to move on the same plane. We also use Morse theory to investigate the number of central configurations, but in this case some difficulties arise from the symmetries which act on the manifold on which $V$ is defined.

To be more precise if we call $M$ this manifold we see that the group $O(3)$ acts over it and this action is not free.[1] This implies that $M/O(3)$ fails to be a manifold and we apply, instead of the classical Morse theory, the equivariant version. In this theory the main tools are to compute the equivariant homology of the manifold $M$ (for the definition see section 2) and to know the isotropy groups of the critical points of $V$.[2]

Actually, in our case, it is just the difference between the isotropy group of the collinear configurations and that of the other configurations that allows us to get some information about the critical points of $V$. In this way we obtain a first estimate of the minnimal number of central configuration. Then, using also the information which comes from the planar problem we get some better results and we are able to say that (for $N > 4$) there are some central configurations such that not all the bodies lay in the same plane.

We would like to thank C. Conley for his encouragement in this research and E. Fadell for many useful talks.

---

(1) We say that the action of a group $G$ on a space $X$ is free if $gx \neq x$, $\forall x \in X$, $\forall g \in G$, $g \neq 1$.

(2) If $x \in X$, the isotropy group $G_x$ in $x$ is: $G_x = \{g \in G : gx = x\}$, where $G$ is the group which acts on $X$.

## 1. Preliminaries

Let $q_1,\ldots,q_N \in \mathbb{R}^3$ denote the positions of N bodies with masses $m_1,\ldots,m_N$ respectively, and $X \subset \mathbb{R}^{3N}$ the linear space given by:

$$X = \{(q_1,\ldots,q_N) \in \mathbb{R}^{3N} \mid \Sigma\, m_i q_i = 0\} \quad .$$

If $\Delta = \bigcup_{i<j} \Delta_{ij} \subset X$ is the set of the diagonals $\Delta_{ij} = \{(q_1,\ldots,q_N) \in X \mid q_i = q_j\}$, $1 \leq i < j \leq N$, then $X \setminus \Delta$ is the configuration space.

The potential energy $V(q)$, $(q = (q_1,\ldots,q_N))$ is the real valued function defined by:

$$V(q_1,\ldots,q_N) = -\sum_{i<j} \frac{m_i m_j}{|q_i - q_j|}$$

and the kinetic energy is:

$$E(q) = \frac{1}{2}\, \Sigma\, m_i |\dot{q}_i|^2 = \frac{1}{2} \langle \dot{q}, M\dot{q} \rangle = \frac{1}{2} \langle p, M^{-1} p \rangle$$

where $M = \begin{pmatrix} m_1 I_3 & & \\ & \ddots & \\ & & m_N I_3 \end{pmatrix}$, $\quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\quad \dot{q} = \frac{dq}{dt}$ and $p = M\dot{q}$.

Denoting by $H(q,p)$ the Hamiltonian function:

$$H(q,p) = \frac{1}{2} \langle p, M^{-1} p \rangle + V(q)$$

we have the following differential equations:

$$(1.1) \qquad \begin{cases} \dot{q} = \dfrac{\partial H}{\partial p} = M^{-1} p \\[2ex] \dot{p} = -\dfrac{\partial H}{\partial q} = -\dfrac{\partial V}{\partial q} \end{cases} .$$

We say that $\bar{q} = (\bar{q}_1,\ldots,\bar{q}_N)$ is a "central configuration" if there exists a scalar valued function $\phi(t)$ such that the solution of the problem (1.1) with initial values $(\bar{q},0)$ is in the form $\phi(t)\bar{q}$.

It follows immediately from the definition that if $\bar{q}$ is a central configuration then $C\bar{q}$ is also for any $C \in \mathbb{R}$. Therefore we may assume that each central configuration belongs to the "mass ellipsoid":

$$\mathcal{E} = \{q \in X : \langle q, Mq \rangle = 1\} \quad .$$

Then it is easy to see that the critical points of $V(q)$ restricted to $\mathcal{E} \setminus (\mathcal{E} \cap \Delta) = \mathcal{E} \setminus \Delta$ correspond in a $1-1$ fashion with the central configurations. So the problem of finding central configurations is equivalent to looking for the critical points of $V(q)$ over $\mathcal{E} \setminus \Delta$.

2. Morse inequalities

The aim of this section is to explore the critical points of $V(q)$ on $\mathcal{E} \setminus \Delta$ using Morse theory.

A first thing to be observed is that the fact that $\mathcal{E} \setminus \Delta$ is not compact is not an obstacle because, since $V(q)$ goes to $-\infty$ in each point of $\Delta$, the critical points of $V(q)$ are bounded away from the set $\Delta$ (for more details see [8]).

Then we consider the symmetries which act on $\mathcal{E}$ and which are given by the diagonal action of the group $O(3) = \{\text{matrices } \Theta \mid \Theta^{\tau}\Theta = id\}$.

Here diagonal action means:

$$\Theta \cdot q = (\Theta q_1, \ldots, \Theta q_N) \quad q \in \mathcal{E},\ \Theta \in O(3) \quad .$$

At this point we observe that the set $\mathcal{E}$ is homotopically equivalent to the sphere $S^{3N-4}$ on which $O(3)$ acts diagonally leaving it invariant.

The potential function $V(q)$ is also invariant with respect to the action of $O(3)$ and $\Delta$ as well. So, because we are interested in the topological-algebraic structure of $\mathcal{E} \setminus \Delta$ we can consider the problem in the set

$$S = \frac{S^{3N-4} \setminus \Delta}{O(3)} = \frac{M}{O(3)} \quad .$$

But in this case the action of $O(3)$ on the manifold $M$ is not free. In fact, there are no fixed points by this action, but the isotropy group is $S^1$, if the configuration $(q_1,\dots,q_N)$ is such that all the bodies are on the same line, instead it is the identity if this does not happen, that is if there exist i, j such that $q_i$ is not parallel to $q_j$ and $q_i \neq 0 \neq q_j$.

This implies that the space $\frac{M}{O(3)}$ fails to be a manifold and we can use, instead of the classical Morse theory, the equivariant version which is obtained by replacing the homology function $H_*$ by the equivariant homology function $H_*^{O(3)}$. This extension of Morse theory involves finding a contractible space U on which the group O(3) acts freely and computing the homology of the space $\frac{M\times U}{O(3)}$ which is a manifold because the action of O(3) is free over the product $M \times U$. Then the Morse inequalities become:

$$M_t^{O(3)}(V) = P_t^{O(3)}(M) + (1+t)Q_t(V) \tag{2.1}$$

where $Q_t(V)$ is a polynomial with positive coefficients, $P_t^{O(3)}M$ is the Poincaré series which represents the homology of $\frac{M\times U}{O(3)}$ and $M_t^{O(3)}(V)$ is the Morse series given by:

$$M_t^{O(3)}(V) = \sum_Z t^{\lambda_Z} P_t^{O(3)}(Z) \tag{2.2}$$

where Z is a critical orbit of V, $Z = O(3)/H$, and $\lambda_Z$ is the dimension of the unstable manifold $\nu^+Z$, that is the dimensionn of the part of the normal bundle $\nu Z$ spanned by the positive eigen-directions of the Hessian of V. So, as first step we need to know the equivariant O(3) - homology of the manifold M which is given by the homology of the space $\frac{V_{\infty,3}\times M}{O(3)}$, where $V_{\infty,3} = \bigcup_{n>3} V_{n,3}$ is a contractible space which is the union of the (orthonormal) 3-frames in $\mathbb{R}^n$ $(n>3)$ and the action of O(3) is free on it (see [4]).

In order to do this we have to compute the homology of M. This is done by observing that the space M is homotopically equivalent to the space

$$F_N(\mathbb{R}^3) = \{q_1,\dots,q_N \mid q_i \in \mathbb{R}^3,\ q_i \neq q_j \text{ for } i \neq j\} = \mathbb{R}^{3N}\setminus\Delta \ .$$

In fact, we can construct a fibration:

$$\begin{array}{c} \mathbb{R}^3 \\ \downarrow \\ \mathbb{R}^{3N} \setminus \Delta \\ \downarrow \pi \\ \{(q_1,\ldots,q_N) \in \mathbb{R}^{3N}\setminus\Delta,\ \sum_{i=1}^{3} m_i q_i = 0\} = X\setminus\Delta \end{array}$$

where $\pi$ is the map which carries the center of mass to the origin. Because the fiber is contractible $X\setminus\Delta$ is homotopically equivalent to $\mathbb{R}^{3N-3}\setminus\Delta$ and hence to $M = S^{3N-4}\setminus\Delta$.

The space $F_N(\mathbb{R}^3)$ has been studied in [3] and its homology with any coefficients is the follows:

$$H_*(F_N(\mathbb{R}^3)) = \bigotimes_{k=1}^{N-1} H_*(\underbrace{S^2 v\cdots v S^2}_{k\ \text{times}})$$

where $\otimes$ is the tensor product and $v$ the wedge sum. So, in short the Poincaré series of $F_N(\mathbb{R}^3)$ is:

(2.3) $$P_t(F_N(\mathbb{R}^3)) = P_t(M) = (1+t^2)(1+2t^2)\ \ldots\ (1 + (N-1)t^2) \quad .$$

Then to compute the equivariant homology of $F_N(\mathbb{R}^3)$ (or $M$) we can look at the fibration:

$$\begin{array}{c} F_N(R^3) \\ \downarrow \\ \dfrac{V_{\infty,3}\times F_N(R^3)}{0(3)} \\ \Big\downarrow p \\ G_{\infty,3} = \dfrac{V_{\infty,3}}{0(3)} = B0(3) \end{array}$$

where $p$ is the projection and $G_{\infty,3} = \bigcup_{n>3} G_{n,3}$ is the union of the Grassmann varieties $G_{n,3} = \dfrac{V_{n,3}}{0(3)}$ of 3-dimensional subspaces of $R^n (n>3)$ and is the classifying space of $0(3)$.

Knowing that the homology of $B0(3)$ (see [4]) with rational coefficients is given by the series:

$$P_t(BO(3)) = \frac{1}{1-t^4} = \sum_{m=0}^{+\infty} t^{4m}$$

from (2.3) ([10]) it follows that the equivariant Poincaré polynomial of $F_N(\mathbb{R}^3)$ (and hence of $S^{3N-4}\setminus\Delta = M$) is:

$$(2.4)\qquad P_t(F_N(\mathbb{R}^3)) = \frac{(1+t^2)(1+2t^2)\ \dots\ (1+(N-1)t^2)}{1-t^4} .$$

From (2.2) and (2.4) we can state the following:

THEOREM 2.1. For each system of $N$ bodies, $N > 3$, with masses $m_1,\dots,m_N$ such that the potential energy $V(q)$ is a Morse function, we have:

$$(2.5)\qquad \sum_Z t^{\lambda_Z} P_t^{0(3)}(Z) = \frac{(1+t^2)(1+2t^2)\ \dots\ (1+(N-1)t^2)}{1-t^4} + (1+t)Q_t(V)$$

where $Z$ is any critical orbit for $V$ restricted to $M/0(3)$ and $Z$ has the same meaning as in (2.2).

To conclude this section we want to observe that, because $Q_t(V)$ has positive coefficients, (2.5) represents the equivariant version of the Morse inequalities.

Moreover, if $Z$ is an orbit given by $0(3)/H$, ($H$ is the isotropy group of any point of $Z$) computing the homology $P_t^{0(3)}(Z)$ is equivalent to computing the series $P_t^H(H) = P_t(BH)$, where $BH$ is the classifying space of $H$.

## 3. Main Results

In this section we will use the Morse inequalities to obtain some estimates of the number of the central configurations of $V(q)$, when $V(q)$ is a Morse function.

We begin by observing that from Moulton's results ([9], [11]) we already know that there are $\frac{N!}{2}$ critical points of $V|_S$ given by configurations with the N bodies on the same line.

For each of these configurations the isotropy group is $S^1$, so, according to the remark made at the end of the previous section, the total contribution of these critical points in the Morse series is the following:

$$\sum_{i=1}^{N!/2} t^{\alpha_i} P_t(BS^1) = \sum_{i=1}^{N!/2} \frac{t^{\alpha_i}}{1-t^2} \tag{3.1}$$

where $BS^1$ is the classifying space of $S^1$ whose homology is given by the series $\frac{1}{1-t^2}$, and $\alpha_i$ is the dimension of the unstable manifold $\nu^+ Z_i$ corresponding to each Moulton configuration.

It is better to remark, at this point, that each critical orbit coming from a Moulton configuration is a 2-dimensional manifold given by $O(3)/S^1$.

Instead, for each critical point of $V|_S$ different from these the isotropy group is the identity. Hence its contribution in the Morse polynomial is given just by a term like $t^{\lambda_Z}$, where $\lambda_Z$ is the dimension of the unstable manifold of the 3-dimensional critical orbit $Z$, corresponding to it, which looks like $O(3)$.

Now, we compute the numbers $\alpha_i$ of (3.1).

PROPOSITION 3.1. _If_ $q = (q_1,\ldots,q_N)$ _is a critical point of_ $V|_M$ _given by a collinear configuration, then the dimension of the unstable manifold of its orbit_ $Z = \frac{O(3)}{S^1}$ _is equal to_ $2N-4$.

_Proof_. Consider the submanifold $Y$ of $M$ defined by the collinear configurations. This represents a submanifold of $M$ of dimension $N-2$.

If $q$ is a critical point of $V|_M$, belonging to $Y$, then the Hessian of $V|_M$ is negative definite on the tangent space $T_qY$ which is $N-2$ dimensional.

On the other hand the Hessian of $V|_M$ is positive definite in each direction normal to Y. Then, recalling that M is 3N-4 dimensional and that the orbit Z of q is a 2-dimensional manifold in M, we get the assertion.

For the planar problem there is not a precise estimate of the exponent $\lambda_Z$, but using the results of [5] we can obtain a lower and an upper bound.

PROPOSITION 3.2. <u>If</u> $q = (q_1,\ldots,q_N)$ <u>is a critical point of</u> $V|_M$ <u>given by a planar configuration, then for its critical orbit</u> $Z = O(3)$ <u>we have</u>:

$$N-3 \leq \lambda_Z \leq 2N-5 \quad . \tag{3.2}$$

<u>Proof</u>. The submanifold X of M given by the planar configurations has dimension 2N-3.

This submanifold is invariant with respect to the diagonal action of $S^1$, (the group of rotations in the plane), and this action is free over X.

So, to every planar central configuration there corresponds a 1-dimensional critical manifold Z' for the potential defined on X.

On the other hand to each planar central configuration there corresponds also a 3-dimensional critical manifold Z" for the potential defined on M which is 3N-4 dimensional.

In each direction normal to X the Hessian of $V|_M$ is positive definite.

Moreover, (see [5]), the Hessian of $V|_X$ is negative definite in at least N-2 directions normal to Z'. So there are at least 3N-7 - (2N-4) = N-3 directions in which the Hessian is positive definite and these directions cannot be more than 3N-7 - (N-2) = 2N-5. From this (3.2) follows.

Now, using these two Propositions and (3.1) and (2.5) we are able to give some estimates of the number of central configurations of N-bodies.

First of all from (3.1), (2.5) and Proposition 3.1, we can rewrite the Morse inequalities in this way:

$$\sum_z \gamma_{\lambda_z} t^{\lambda_z} + \frac{N!}{2}\frac{t^{2N-4}}{1-t^2} = \frac{(1+t^2)(1+2t^2)\cdots(1+(N-1)t^2)}{1-t^4} +$$

$$\text{(3.3)} \qquad + (1+t)Q(t) = \frac{\sum_{0\leq i\leq N-2} \beta_i t^{2i}}{1-t^2} + (1+t)Q(t)$$

where $0 \leq \lambda_z \leq 3N-7$, $\gamma_{\lambda_z}$ is the number of critical orbits (which do not come from collinear configurations) with dimension of the unstable part of the normal bundle equal to $\lambda_z$ and $\beta_i$ are the Betti numbers.[3]

Note that $\sum_{i=0}^{N-2} \beta_i = \frac{N!}{2} \qquad N \geq 3.$

From (3.3), recalling that $Q(t)$ has positive coefficient we can deduce the following:

THEOREM 3.1. Let $m_1,\ldots,m_N$ be the masses of N-bodies such that the potential energy $V(q)$ is a Morse function. Then, if $\gamma_{2i}$ is the number of critical orbits of $V|_S$ whose unstable manifold has dimension $2i$, we have:

$$\text{(3.4)} \qquad \gamma_{2i} \geq \beta_1 + \beta_2 + \cdots + \beta_i \qquad 0 \leq i \leq N-2$$

---

(3) The numbers $\beta_i$ can be computed by this formula:

$$\beta_i = (-1)^i \sum_{K=0}^{i} S_N^{N-K}$$

where $(-1)^{p-q}S_p^q$ is the number of permutations of $p$ elements with $q$ cycles. The numbers $S_p^q$ are called Stirling numbers of the first kind.

where $\beta_j$ are the Betti numbers previously defined.

(3.4) gives a first estimate of the minimal number of critical points that $V(q)$ has on $M$.

An important consequence of (3.4) and Proposition (3.2) is that, whenever $2i < N-3$, the configuration whose orbit has the dimension of the unstable manifold equal to $2i$ cannot be planar but need to be spatial.

The number of these configurations increases as $N \to +\infty$.

For example, for $N=4$, we can say that there exists at least one configuration whose orbit has the dimension of the unstable manifold equal to $0$ and which cannot have all the four masses positioned on the same plane. This configuration corresponds to a tetrahedron.

For $N=6$ we can say that there is at least one spatial configuration whose orbit has the dimension of the unstable manifold equal to $0$ and at least 16 different spatial orbits for which this dimension is equal to $2$.

Now, without going into details, we want to mention some other consequences which come from (3.3) and Proposition 3.2 and which improve Theorem 3.1.

We know from the results of [5], [6], [7] what the minimal number of planar central configurations is and what their indexes are.(4)

Then, knowing that in each direction normal to $X$ the Hessian of $V|_M$ is positive definite, we can compute the right exponent that each of these configurations carries into (3.3). Moreover, considering the action of $O(3)$ on these planar configurations, we can compute their effective number as

(4) If $X$ is the submanifold of $M$ given by the planar configurations, the index of $V$ restricted to $X/S^1 = \bar{X}$ in a point $q \in \bar{X}$ is the maximal dimension of the subspace of $T_q\bar{X}$ on which the Hessian of $V$ is negative definite.

orbits on the manifold $M$. So doing, and considering also (3.4) we can see that, in order to find a polynomial $Q(t)$ with positive coefficients which satisfies (3.3), there must be other configurations different from those already computed.

For example for 5 equal masses we discover that there is at least one configuration whose exponent in (3.3) is equal to 1 and which, in virtue of Proposition 3.2 $(N-3=2)$ cannot be planar. To explain this idea better we conclude by examining the case of 4 equal masses.

From [6] we know that, for the problem on the plane, there are exactly 146 classes of relative equilibria, given by 12 Moulton classes with index 2, 6 square configurations with index 0, 8 equilater triangles with a mass at each vertex and the 4th mass at the center, with index 2, 24 isosceles configurations with a mass at each vertex and another one in the interior on the axis of symmetry, 96 given by two pair of scalene configurations with a mass at each vertex and one in the interior.

Except for the Moulton case, when we consider these classes of relative equilibria as critical orbits on $M$, we see that, because of the action of $O(3)$, their number is just half of the previous one. Moreover, the number of unstable directions for each of these orbit increases of 1 going from the planar problem to the space.

The number of unstable directions for each Moulton critical orbit is $2N-4=4$. So we have 27 critical points of $V|_S$ with exponent 1, 36 with exponent 2, 4 with exponent 3 and 12 with exponent 4.

Filling in (3.4) with these numbers we obtain that there exist 2 other different critical orbits with exponent 0 which correspond to a unique spatial configuration given by a regular tetrahedron.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper we use topological methods, in particular Morse theory, to study the problem of finding spatial central configurations of the N-body problem in $\mathbb{R}^3$. The principal difficulty in applying Morse theory is that the potential function is defined on a manifold on which there is the action of a group which is not free. This suggests using the equivariant homology functor in order to obtain the Morse inequalities which enabes us to obtain an estimate of the minimal number of spatial central configurations.

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